Symmetries of Systematic Errors Due to Berry's Phase

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December, 2003

Numerical calculation of some of this class of systematic errors has been done. This note studies the symmetry properties of these effects. Ref. 1 discusses the second order effect of local non-cancellation of ω_a in the presence of a local longitudinal magnetic field which are spacially out of phase. In this note I study the symmetry of this error via the simple process of rotation by φ about the y axis followed by rotation of θ about the z axis:

$$x' = x\cos\varphi + z\sin\varphi$$
$$z' = z\cos\varphi - x\sin\varphi$$

$$x' = x\cos\theta + y\sin\theta$$
$$y' = y\cos\theta - x\sin\theta$$

The sign of φ and θ is reversed, and the procedure is repeated. The problem arises because rotations do not commute. Table 1 summaries the results for the case when the signs of φ and θ were alternated 10^5 times each.

φ	θ	initial x	initial y	initial z	final x	final y	final z
$\pm 10^{-3}$	$\pm 10^{-3}$	0	0	1	0	+0.05	0.9988
$\mp 10^{-3}$	$\mp 10^{-3}$	0	0	1	0	+0.05	0.9988
$\mp 10^{-3}$	$\pm 10^{-3}$	0	0	1	0	-0.05	0.9988
$\pm 10^{-3}$	$\mp 10^{-3}$	0	0	1	0	-0.05	0.9988

The first two entries correspond to rotations by:

$$+\varphi \quad +\theta \quad -\varphi \quad -\theta \ldots \quad (1)$$
$$-\varphi \quad -\theta \quad +\varphi \quad +\theta \ldots \quad (2)$$

which give the same result: a final positive y component. One can see that 2) is the same pattern as 1), just displaced in time. From ref. 1: "It is amazing that the spin rotates around the radial, not the longitudinal axis as a result of the combination of the oscillating rotations around the longitudinal axis and the oscillating rotations around the vertical axis".

Fig. 1 shows what happens step by step. Step 0 is the initial state: x = y = 0, z = 1. First we rotate about the y axis, so x increases by $\varphi z \approx \varphi$, but y does not change. Then we rotate about the z axis, so y increases by $\theta x \approx \theta \varphi$, and x decreases by a very small amount $\approx \theta^2/2$. Then we rotate about the -y axis, so x comes back close to zero, and y remains the same. Then we rotate about the -z axis, and both x and y change by a very small amount. Then we repeat. The y component keeps growing.

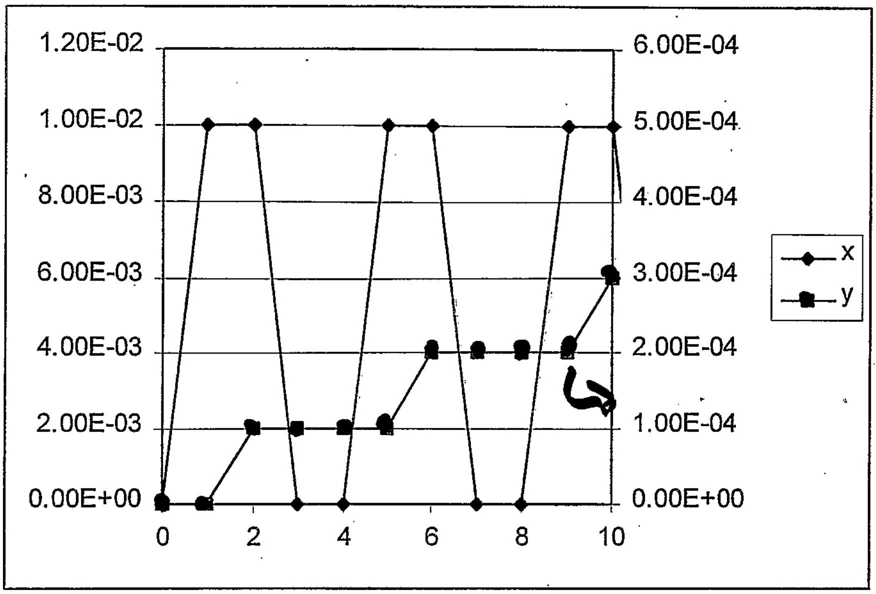


Fig. 1. Rotations about the y and z axis by 0.01. The scale for the x component is on the left, and the y is on the right. The z component remains close to 1.

The last two entries in Table 1 correspond to rotations by :

$$-\varphi \quad +\theta \quad +\varphi \quad -\theta \ldots \quad (3)$$
$$+\varphi \quad -\theta \quad -\varphi \quad +\theta \ldots \quad (4)$$

which give a final negative y component. 1) corresponds to a different physical phase of φ and θ compared to 3). Fig. 2 shows one of these two situations, assuming the E field is "perfect", ie. completely uniform around the ring. The other physical situation occurs when $|B_0|$ too low and too high are interchanged.

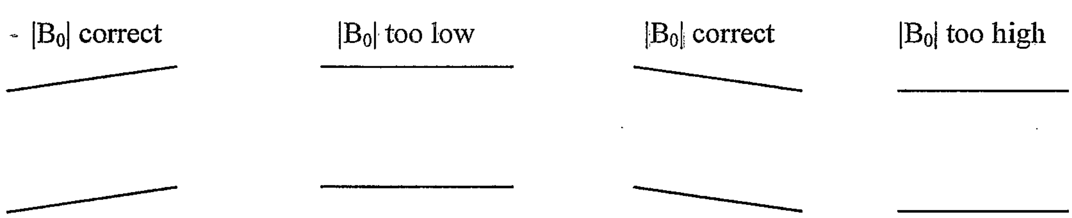


Fig. 2. This combination of magnet tilts and incorrect magnitudes causes a Berry's phase systematic error [1].

The tilted magnet gives a longitudinal component of the magnetic field. By Ampere's law, if there are no enclosed currents, the net longitudinal component of the magnetic field is zero around the ring. If the magnitude of B is incorrect in the tilted magnets, it doesn't matter for the Berry's phase systematic error, as the φ and θ rotations then happen simultaneously. Now let's consider what happens when we inject clockwise and counter-clockwise.

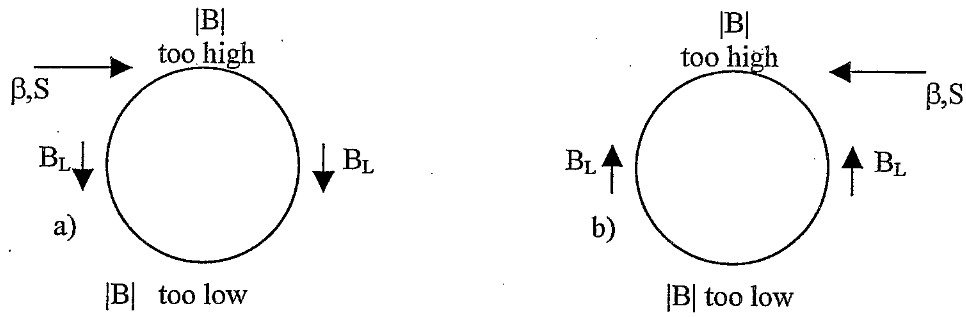


Fig. 3. a) Clock-wise and b) counter-clockwise.

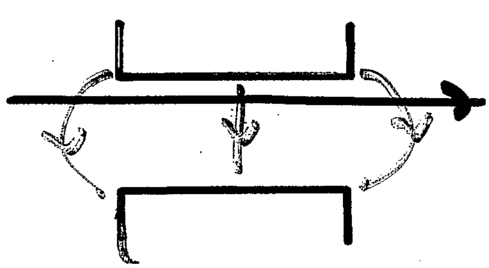
For both cw and ccw the spin gets rotated radially inward when the magnetic field is too high compared to the electric field (for positive anomaly). For clockwise, the spin next gets rotated upward by the longitudinal magnetic field in Fig. 3a: $dS/dt = \mu \times B$. For counter-clockwise, the spin also gets rotated upward upward by the longitudinal magnetic field in Fig. 3b. This is the opposite symmetry as for the edm, ie. if the spin gets rotated upward by the edm when the beam is going cw, it gets rotated downward when the beam is going ccw: $dS/dt = d \times (\beta \times B)$. This is in agreement with ref. 2; it's always good to check signs.

One can also get an effect if the beam is too high, for example if the quadrupole magnets are higher than the dipole magnets, so that the fringe field has an average positive longitudinal component as the beam enters the magnet, and an average negative longitudinal component as the beam leaves the magnet. This gives rotations about:

$$+\theta \quad +\varphi \quad -\theta \quad +\theta \quad -\varphi \quad -\theta \ldots$$

Running the program on this configuration gave no effect (for obvious symmetry reasons).

Another Berry's phase systematic error occurs if the average E and B fields are not matched in addition to local longitudinal magnetic fields. Fig. 3 shows the case for initial $x = -1$, $y = z = 0$, 1,500 operations of $\varphi = 0.001$ and $\theta = \pm 0.001$. The final rotation about the y axis is 1.5 radians. At the beginning, $x = -1$, so rotation about the z axis by $\pm 10^{-3}$ gives $y = 0$ or 10^{-3}. The average is 5×10^{-4}. At the end, x is small and y is small, so rotations about the z axis gives very little change. The average is still 5×10^{-4}, so there is no change in the average. However, in reality, a polarimeter measures the spin at one location in the storage ring. Hence, it could see either 0 or 10^{-3} at early times and 5×10^{-4} at late times, which is an observational systematic error. This argues for many polarimeters around the ring.



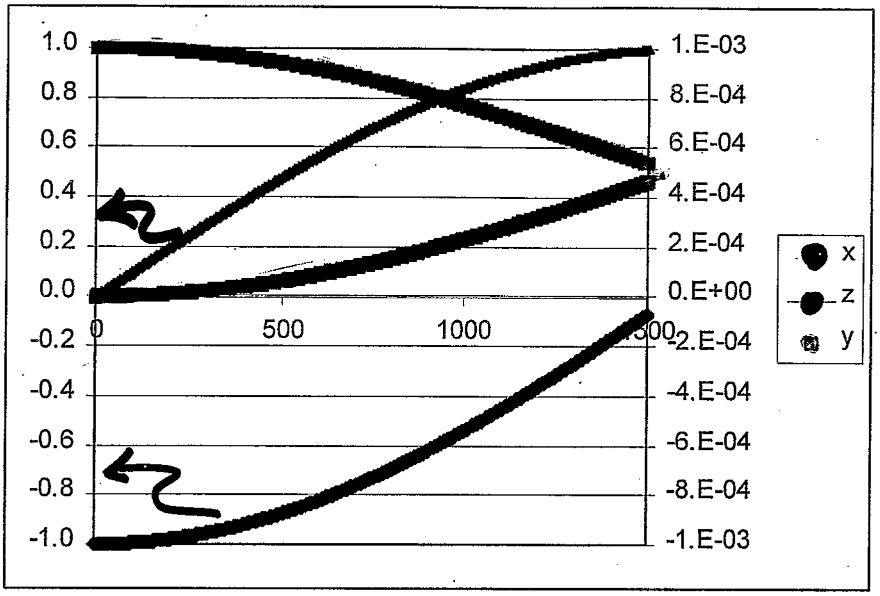


Fig. 3. 1,500 operations of $\varphi = 0.001$ and $\theta = \pm 0.001$. y scale is on the right.

References

1. Y. Orlov, EDM Note 26.

2. Y. Orlov et al., submitted as EDM Note.